THE APPLICATION OF c-ϕ REDUCTION METHOD TO ESTIMATE THE BEARING CAPACITY OF SUBSOIL

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Received: 7.11.2013; Revised: 15.12.2013; Accepted: 15.12.2013

Abstract

The paper presents the method for estimation of the bearing capacity of a subsoil under a foundation, which uses the c-ϕ reduction procedure applied in stability analyses. The proposed computation method enables analysing a foundation loaded in any way at any arrangement of soil layers, which frequently cannot be taken into account in classical considerations using traditional analytical solutions available in standards. Within performed analyses the results obtained with the use of the presented method were compared with the solution presented in Eurocode 7 for various foundation cases. A very good consistency of numerical results with analytical solutions was achieved. The paper comprises also estimation of subsoil bearing capacity under a track superstructure founded on an embankment.

Streszczenie

W artykule przedstawiono metodę szacowania nośności podłoża gruntowego pod fundamentem, która wykorzystuje procedurę redukcji c-ϕ, która wykorzystywana jest w analizach stateczności. Zaproponowana metoda numeryczna umożliwia analizę zachowania się fundamentu obciążonego w dowolny sposób przy dowolnych warunkach gruntowych, co zwykle nie jest możliwe przy stosowaniu tradycyjnych analitycznych metod proponowanych przez normy. Wyniki rozwiązanych z wykorzystaniem prezentowanej metody zagadnień różnych typów fundamentów zostały porównane z wynikami uzyskanymi na podstawie normy Eurocode 7. Uzyskano bardzo dobrą zgodność rezultatów obliczeń numerycznych z wynikami rozwiązań analitycznych. W artykule zaprezentowano również oszacowanie nośności podłoża pod nasypem wraz z wykonaną na nim nawierzchnią kolejową.

Keywords: **Bearing capacity of subsoil; Stability analysis; FEM.**

1. INTRODUCTION

To be capable of estimating the bearing capacity of the subsoil under a foundation it is necessary to adopt an appropriate model of subsoil operation together with the structure founded on it. Based on that, having carried out calculations, the value of a load resulting in reaching the structure bearing capacity – in this case the subsoil under a foundation – is obtained. Analyses carried out in the past allowed obtaining an analytical solution in a closed form (e.g. formulae available in standards), while now numerical methods are more and more often used to perform such analyses. The latter ones enable using much more complex constitutive models of the subsoil and also taking into consideration various effects and properties affecting the obtained results. The available computer software enables performing quick computations and achieving more precise results, which can accurately represent the reality. The problem related to the knowledge of used models assumptions (a numerical software is frequently treated as a "black box"), and primarily of selecting proper parameter values for calculations, appears here. Frequently a situation occurs that the performance of advanced numerical analyses is accompanied by the selection of model parameters based on an old standard or references, and the soils studied are entirely different than those, for which simulations are carried out. A situation may result then in a poor accuracy of complex computations resulting from the uncertainty of parameter values estimation, which frequently exceed 50% of their real value. However, despite of that it is important to start studies on creating simple computational methods and procedures, which will allow proper estimation of subsoil bearing capacity in various geo-engineering problems.

The paper presents the application of c-ϕ reduction method to estimate the bearing capacity of subsoil under a foundation, which is frequently used in FEM analyses of slopes and hillsides stability. The cases considered refer to various cases of spread foundation and take into account various factors affecting the obtained results.

2. METHOD OF c-ϕ REDUCTION IN NUMERICAL ANALYSES

Computational models, which allow estimation of bearing capacities of various structure types, to a large extent use an elasto-plastic description of material behaviour, in this case the soil medium. An assumption is made that – after reaching by a point in a stress space of a certain plasticity surface – the material's response to the applied influence features an unlimited increase of deformations, with the lack of capability to accept higher loads. Such behaviour of the medium may be defined as perfectly plastic and in the case of uniform state of stress it will determine the structure failure. In typical soil mechanics problems the stress state in an element or in a soil medium is variable and therefore reaching the strength at the given point will not result in the whole structure failure, but it will determine the beginning of a stress redistribution process. So in other words, in the case of softening caused at one point (or within an element), the additional load will be taken over by the neighbouring elements. Such a situation will be possible as long as a certain destruction mechanism is not created in the whole structure, after reaching which the structure loses a possibility of transferring any additional load. This way the bearing capacity of the analysed structure is estimated. In practical problems both the load meaning the reaching of structure bearing capacity (to show that the actual influence, when using appropriate safety coefficients, is lower than the bearing capacity) and a failure mechanism are sought. This last piece of information will be necessary when selecting methods of strengthening or possibilities of taking over an additional load at other influence schemes.

One of computational methods allowing to analyse failure mechanisms of a subsoil is the method of c-ϕreduction of the soil medium strength. It is usually used to analyse slopes stability together with buildings founded on them as well as to assess the operation of retaining structures [1], [2]. However, this method use may be much wider. In a general approach the c-ϕ reduction method allows generating a failure mechanism, which results from the analysis of a stress state in a soil environment. The procedure of c-ϕ reduction consists in a gradual decreasing of strength parameters values, which translates into a change of the stress state at individual points of the analysed medium, increasing the softening areas and finally detects the origination of a failure mechanism. In the case of stability analyses this allows estimation of the stability coefficient [3], however, in a general case it can allow specifying the bearing capacity. A soil medium together with a building foundation founded on it is a structure here.

Calculations using the procedure presented at the beginning assume performance of the analysis of soil medium response to the applied influence. After the generation of primary stresses a load is applied, most frequently in appropriately selected incremental steps. The analysis performed should reflect implementation stages of actual works carried out on the construction site. The stress distribution in a soil medium is determined based on the course of simulation, taking into account the operating loads, executed excavations, anchoring and supporting elements etc. An elasto-perfectly plastic constitutive model with a Coulomb – Mohr plasticity surface is most often used in computations. The c-ϕ reduction procedure may be performed at any stage of computations. During computations, in steps assumed by the user, strength parameters are decreased acc. to the relationship:

$$F_i = \frac{c_0}{c_i} = \frac{tg\phi_0}{tg\phi_i}, \tag{1}$$

where c_0 and ϕ_0 – are values of cohesion and friction angle for individual soil layers at the beginning of the analysis, c_i and ϕ_i – are values of strength parameters in the i-th computational step, F_i – is the current value of the reduction coefficient.

The procedure described above is carried out till the moment of reaching computations non-convergence. During calculations, as a result of deterioration of a soil medium strength properties, softening occurs in a larger and larger number of points (achieving a corresponding point of plasticity surface in the stress space). Iterations performed cause a change of the stress state, allowing their redistribution. However, at the moment there is no further possibility of softening states iteration and the computation process becomes a divergent process. The value of coefficient F and a corresponding failure mechanism are obtained as the result of analyses. The analysis may be performed at any stage of considerations. Very often it is performed many times during the resolution of a set problem. Coefficient F may be understood as a safety reserve at a given stage of numerical analyses. It is necessary to emphasise here that the obtained result frequently depends on the effectiveness of the adopted scheme of constitutive equations integration and on the iteration algorithm. Therefore an accurate analysis of the failure surface, allowing a reliable assessment of the obtained safety reserve, is equally important, apart from obtaining the looked for coefficient F.

3. SUBSOIL BEARING CAPACITY IN AN APPROACH OF STANDARDS

Problem of the estimation of the bearing capacity of the soil was solved by Trzaghi [4]. Complying to them failure mechanism is shown in the figure 1. It assumes that at the moment of reaching the subsoil bearing capacity a rigid wedge will be created in the soil environment directly under the foundation, which will cause displacement of soil masses situated next to the analysed foundation. A model of elastic-plastic half-space was used in the analysis and the soil environment behaviour was described by a Coulomb – Mohr model. In the case of high values of friction angle, the originated failure mechanism will be accompanied by displacement of large soil volumes (cf. Fig. 1), which will directly translate into high values of obtained bearing capacities.

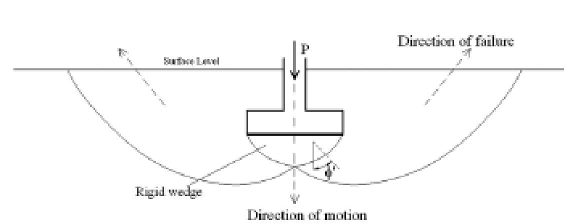

Figure 1.
Terzaghi's failure mechanism of subsoil under the foundation

A formula allowing estimation of the bearing capacity of subsoil under the foundation base taken by Terzaghi solution was present in the Polish standard [8]. Similar form is also adopted in the Eurocode 7 [7]. According to the latter document the load bearing capacity expressed in stress units is given as:

$$R/A' = c \cdot N_c \cdot b_c \cdot s_c \cdot i_c + \gamma_D \cdot D \cdot N_q \cdot b_q \cdot s_q \cdot i_q + \gamma_B \cdot B \cdot N_\gamma \cdot b_\gamma \cdot s_\gamma \cdot i_\gamma . \qquad (2)$$

In the above formula R/A' stands for the bearing capacity of subsoil under the foundation, expressed as a ratio of force to effective surface area (having considered eccentrics of the vertical force loading the foundation in both directions). As it can be seen, this bearing capacity depends primarily on the value of internal friction angle, on whose values of bearing capacity coefficients N_c, N_q and N_g depend. The other factors affecting a possibility of loads transfer by the subsoil consist of cohesion c for cohesive soils, foundation depth D and the weight of soil above the foundation level γ_D as well as a smaller foundation size (width) B and the weight of soil below the foundation level γ_B. If groundwater exists within the foundation level, the above weights should be reduced by the buoyancy of water. The next coefficients in formula (2): b, s, i stand for the influence on the bearing capacity of the foundation base slope, foundation shape and the slope of resultant load, respectively. Formula (2) applies to the operating conditions of so-called "drained" subsoil, pretty well simulating operation of typical foundations, which usually are loaded slow enough that an excess of pore pressures will not be generated in the subsoil. This relationship is commonly used by designers at checking conditions of the bearing capacity state at foundations designing. Bearing capacity of the subsoil estimated by (2) will be used to verification of the result obtained on grounds of proposed c-φ reduction method.

4. THE APPLICATION OF c-φ REDUCTION METHOD TO ESTIMATE THE BEARING CAPACITY OF SUBSOIL UNDER THE FOUNDATION

As it has been mentioned in section 2 of this paper, having carried out the analysis of c-φ reduction, a certain failure mechanism corresponds to the obtained value of the bearing capacity coefficient F. In the case of most analyses of slopes stability this mechanism consists in the formation of a landslide

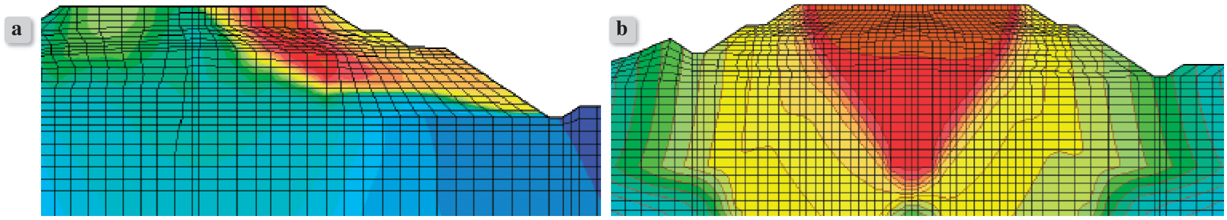

Figure 2.
Failure mechanism obtained by stability analysis: a) landslide form, b) reaching the bearing capacity

along some failure surface (Fig. 2a). However, it turns out that in some cases (Fig. 2b) the originated failure mechanism resembles a form of subsoil bearing capacity loss shown in Fig. 1. The formation of a rigid wedge under the loaded surface is visible, and the reaching of subsoil strength is accompanied by the displacement of soil masses situated next to the loaded area. It means that the applied c-ϕ reduction method in this case gives a failure mechanism corresponding by its nature to a loss of subsoil bearing capacity under the foundation and not to a slide along certain surface, which is expected by default when using the considered computational procedure.

Performing calculations with the use of the c-ϕ reduction method, while the coefficient F increase, the soil resistance is reduced. It is done at a given point by the equation:

$$\tau_f = \sigma \cdot \mathrm{tg}\phi + c, \tag{3}$$

where τ_f – shear resistance (maximal possible shear stress, which could be taken by soil medium), σ – normal stress (to the direction of the future slide surface). Reaching of the soil resistance according to the condition (3) causes activation the failure mechanism, which responds to reaching the bearing capacity of the subsoil. That failure mechanism arises along particular surface (line in the plan case).

Taking into account the above statements it is possible to notice that the c-ϕ reduction method may also be used to estimate the bearing capacity of subsoil under any load, also under a foundation. The value of bearing capacity coefficient obtained from calculations means a safety reserve featured by a given structure subject to specific influence. So it is possible to assume that the subsoil bearing capacity under a foundation may be determined based on the relationship:

$$R = P \cdot F, \tag{4}$$

where: R – bearing capacity of subsoil under a foundation, P – load acting on a foundation, which was used for calculations, F – the value of reduction coefficient obtained from computations. Because analysed problem is nonlinear, obtained result could depend on used in (4) the load value P. Hence it suggests, that received value F should not exceed 2.0. Otherwise the calculations must be repeated for greater P, which guarantees obtaining results close to the real value.

It is worth adding that the way of bearing capacity estimation based on formula (4) gives much greater possibilities of application in problems of foundations behaviour analyses. When performing calculations by means of the finite elements method and using the procedure of c-ϕ reduction it is possible to analyse much more complicated structures at the existence of diverse soil conditions, complex loads and load paths at staging the considered problem and many other soil engineering issues.

5. ASSUMPTIONS OF PERFORMED NUMERICAL ANALYSES

To check the application possibilities of previously presented computational method in practical engineering problems, the bearing capacity of subsoil under a foundation for five selected computational variants will be estimated further on. Four of them will refer to 2D analyses (assuming a plane strain state), and the fifth to a 3D case. The obtained results will be compared with values obtained based on relationship (2), which is used in typical design calculations. Diagrams of four considered plane problems are presented in Fig. 3. The first case is an axially loaded strip foundation (and more precisely its 1 m long section), which is founded on a soil of medium bearing capacity (clay of plastic consistency). The second considered variant is the case of a force acting on an eccentric, while the third – the existence of additional horizontal force, whose value is 20% of the vertical force. Both these foundation variants were considered on the subsoil identical with the first case variant. The fourth of analysed problems (Fig. 3d) is

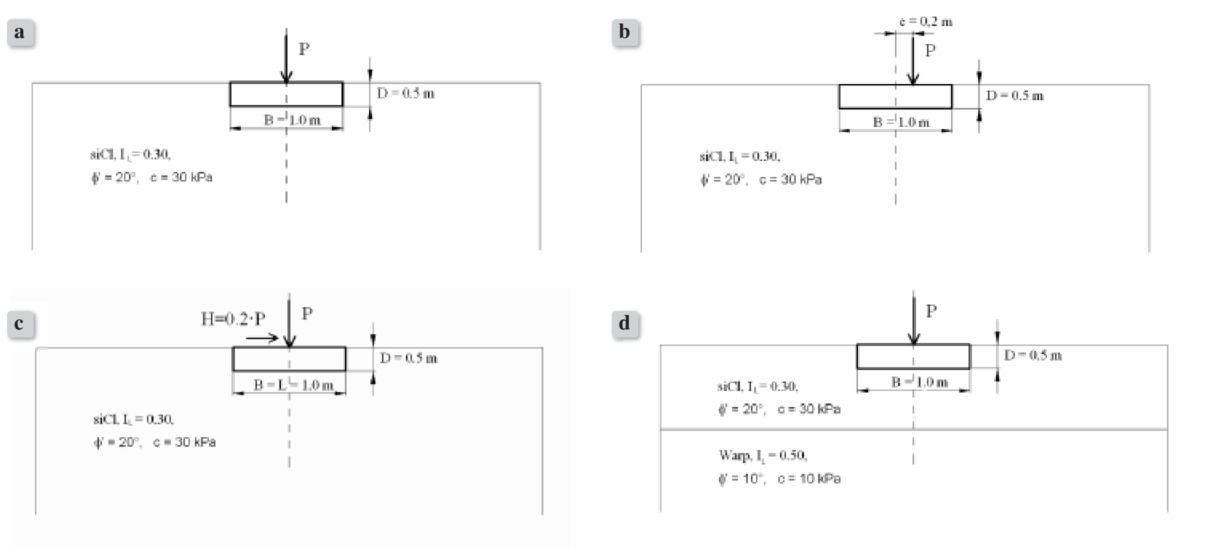

Figure 3.
Analysed calculation variants: a) force in the middle of foundation, b) eccentricity force, c) influence of horizontal force, d) layered subsoil

the case of weak soil existence (warp of soft consistency) at the depth of 2.0 m below the ground level (1.5 m below the foundation base level). In all analysed cases foundations were founded at the depth of 0.5 m below the ground level and were $B = 1.0$ m wide. Computations were carried out using an elasto-plastic constitutive model with a Coulomb-Mohr plasticity surface. Values of strength parameters for individual soils making the subsoil are given in Fig. 3.

Numerical computations were carried out with the use of finite elements method using the Z_Soil 2010 software [5]. Having generated primary stresses in the subsoil (the state analysed was with the constructed foundation, without considering intermediate stages, i.e. the excavation and the strip construction) the load was applied acc. to schemes shown in Fig. 3. Values of load P used in computations are given in Table 1 at the discussion of obtained results. They should be adapted to possibilities of load transferring by individual foundations and first of all – should not exceed the bearing capacity of subsoil under the foundation. The last stage consists of making calculations by means of c-ϕ reduction procedure. The latter ones result in obtaining the bearing capacity coefficient F, for which the end of bearing capacity under the foundation has been obtained. Based on F, using formula (4), the sought after bearing capacity of foundation founded on the considered subsoil is obtained.

The fifth variant analysed in the paper, used to compare results obtained with the use of the suggested method, is the problem of estimating the bearing capacity of the spot footing, of projection dimensions $B = L = 1.0$ m. In this case a 3D problem was resolved. The other assumptions taken for computations and their course are identical with those for the first case of analysed 2D problems.

6. RESULTS OF CALCULATIONS

6.1. Comparison of Numerical Analyses Results with a Solution from Standards

The first stage of analyses consisted in using the proposed method for bearing capacity estimation to compare the obtained results with those obtained with the use of relationship (2) provided in Eurocode 7 standard. The results of analyses are specified in Table 1. Bearing capacities of the subsoil were estimated using formula (4) based on the obtained value of coefficient F. The obtained failure mechanisms corresponding to the end of load bearing capacity of the subsoil under foundation are presented in Fig. 4 and 5. In some figures (4a, 4c and 4d) failure mechanisms, distinguished based on the map of displacements of the analysed system at the moment of achieving non-convergence of the defined numerical problems, are marked with a thick line. The mechanisms with a pretty high precision correspond to those taken for considerations within a classical soil mechanics. In the case of axially loaded strip (Fig. 4a) or spot (Fig. 5) footing this will be the soil displace-

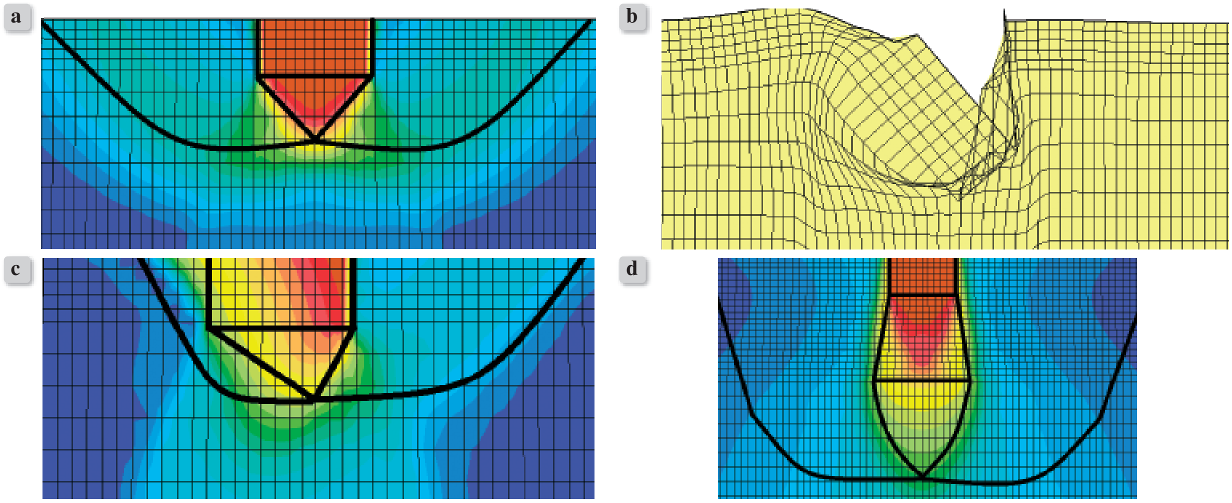

Figure 4.
Failure mechanism obtained from analysed variants: a) force in the middle of foundation, b) eccentricity force, c) influence of horizontal force, d) layered subsoil

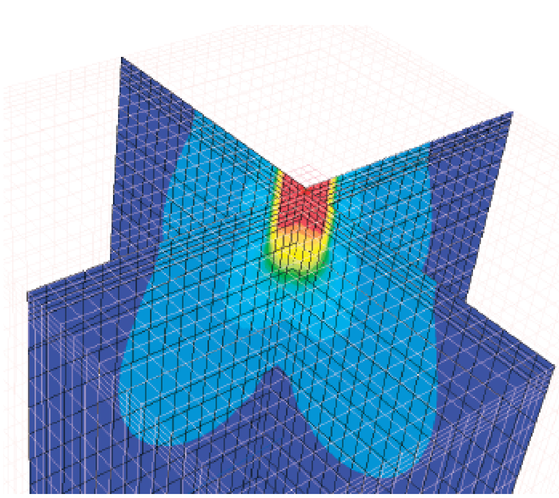

Figure 5.
Failure mechanism obtained from a 3D analysis (spot footing)

ment from under the foundation base, in the case of a week soil layer deposited below (Fig. 4d) – the displacement of strong soil from under the floor, while in the case of a substantial horizontal force existence (Fig. 4c) – an asymmetrical soil displacement. A slightly different situation will occur in the case of a large eccentric of the vertical force (Fig. 4b), where the end of load bearing capacity will be proven by a very high tilt of the foundation and resulting plastic yielding of the soil environment under one of its edges, at simultaneous occurrence of tearing off on the other side. In all considered variants the obtained results (both the estimated values of bearing capacity as well as corresponding failure mechanisms) are consistent with results obtained based on analytical methods provided in standards ([7], [8]). It is also necessary to pay attention to the value of force P taken for calculations. As the proposed computational procedure assumes reducing values of strength parameters of soils deposited in the subsoil, so the

Table 1.
Specification of computation results

Analysed problem	Adopted value of force P [kN]	Obtained coefficient F	Bearing capacity acc. to the c-φ reduction method [kN]	Bearing capacity acc. to Eurocode7 [kN]	Difenrence in results [%]
Axially loaded strip (1 m long)	500	1.24	620	548	+13%
Eccentrically loaded strip (1 m long)	200	1.71	342	329	+4%
Obliquely loaded strip (1 m long)	200	1.91	382	368	+4%
Strip on a layered subsoil (1 m long)	200	1.37	274	243	+13%
Axially loaded footing	400	1.68	672	697	– 4%

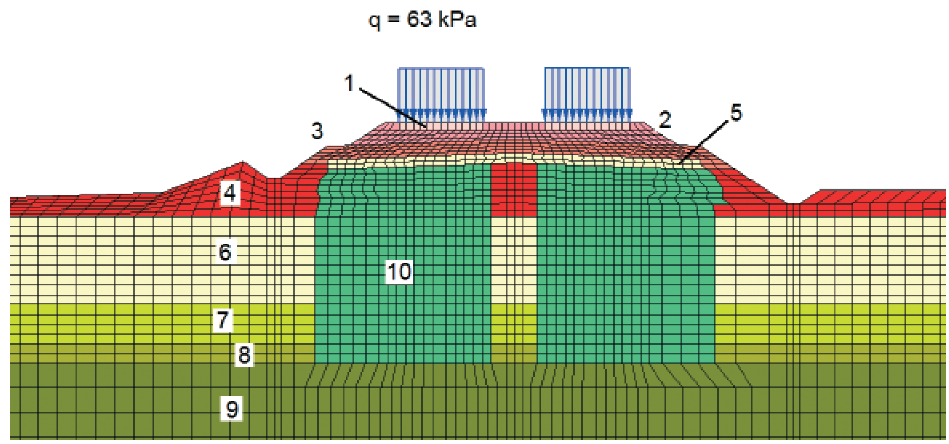

Figure 6.
Boundary problem of the subsoil behaviour under a railway structure

obtained bearing capacity coefficient *F* should not be higher than 2.0 due to the fact that at designing partial reduction coefficients are virtually never higher. Therefore, in the case of improper selection of force *P* value (the obtained value *F* is higher than 2.0 or computations become divergent before applying full value of *P*), it must be appropriately adjusted.

The obtained bearing capacities (Table 1) in most cases are slightly higher than those obtained based on methods from standards and the differences fall within 4-13% tolerance. So it should be considered that for the analysed test problems the proposed method allows obtaining bearing capacity values of subsoil under a foundation close to those resulting from standards.

6.2. Application of the Proposed Method to Estimate the Bearing Capacity

Special attention should be paid to the fact that the considered computational situations, despite very many influences (cf. formula 2), are burdened with certain assumptions, which do not allow resolving analytically of any problem of subsoil under a foundation behaviour and estimating the bearing capacity of so defined structure. The proposed method is free of such limitations. Because – when defining a boundary problem, which is to be resolved using the finite elements method – it is possible to consider any structure of the foundation (diversified shape, anchoring, supporting with piles etc.) founded under any soil conditions (inclinations of soil layers, weak soil lenses) as well as many other factors (founding on a slope, close to excavations etc.), which affect the obtained results. The analysis of such problems is not possible using the relationships from standards.

The behaviour of a railway structure founded on a small embankment 2.5 m high was considered as an example of the title method use in geotechnical problems. Under layers of track superstructure there is a 3.0 m layer of very weak organic soils deposited in firm and soft consistency. Because of that the strengthening of a weak subsoil with driven stone columns was applied [6]. The numerical model of defined boundary value problem describing the problem is shown in Fig. 6, while significant parameters of a Coulomb – Mohr model used in the analysis are specified in Table 2. In the case of stone columns the parameter values were taken applying homogenisation resulting from the assumption of a plane strain state (strong material of columns inside weak layers of organic soils). Projection of column centres creates a grid of equilateral triangles with a common base – hence their model adopted in Fig. 6 results on the basis of a homogenisation assumption. Non-zero cohesion values for cohesionless soils were taken based on consideration of interlocking effect (grains jamming) and approximation by a straight line of a non-linear τ–γ characteristic in the field of small stresses.

The analysis of previously described problem consisted in finding primary stresses in the subsoil, loading by an action resulting from the rolling stock movements (q = 63 kPa) and in performing the bearing capacity analysis of so defined structure. When analysing the behaviour of the considered system it was expected that the lack of iteration convergence of the computations will result from the origination of a failure mechanism corresponding to a typical slide on some surface. The instruction [9] requires obtaining a bearing capacity coefficient value higher than 1.5 in

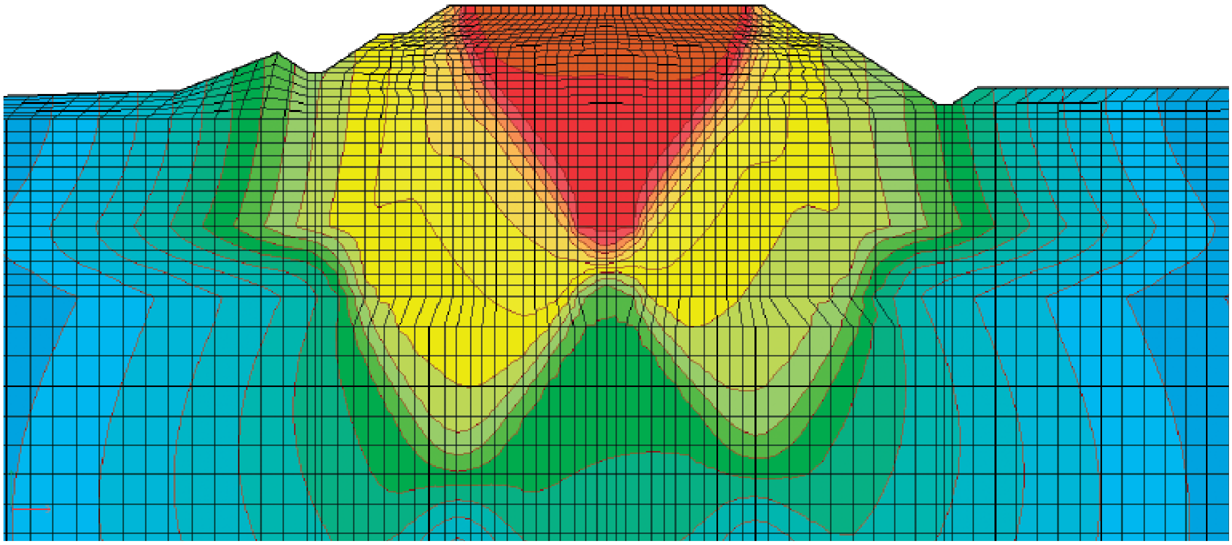

Figure 7.
Failure mechanism for a railway structure founded on a weak subsoil

accordance with rigours applicable to modernised railway lines. However, it turned out that the failure surface (Fig. 7) resembles a soils displacement from under the foundation, hence the load bearing capacity of the subsoil under the railway superstructure was estimated based on calculations. As the obtained value of the bearing capacity coefficient in this case was 1.54, so the bearing capacity of so defined structure can be estimated as:

$$q_f = q \cdot F = 63 \cdot 1.54 = 93 \text{ kPa} \qquad (5)$$

Although the above value is small, anyhow it exceeds the action produced by the rail superstructure on the subsoil. It should be noted here that a very large load from the wheels of moving trains is not considered (such loads are taken over by appropriately designed rail superstructure), but pressures from the rail superstructure on the subsoil, in the light of stability loss risk and the end of the bearing capacity of the subsoil under the structure. The estimation method proposed in the paper is undoubtedly more accurate than classical analysis methods, whose use in the example in question would be related to the necessity to make numerous simplifying assumptions.

Table 2.
Values of parameters taken for numerical calculations

No of material zone	Type and condition of a soil layer	Angle of internal friction φ'[°]	Cohesion c [kPa]	Modulus of elasticity E [MPa]
1	Railway sleepers	30	100	10,000
2	Rail superstructure	40	10	200
3	Rail base	36	5	120
4	Railway embankments	30	2	60
5	Geomattress aggregate	40	10	200
6	Highly organic, soft soils	6	5	6
7	Highly organic, plastic soils	11.5	7	12
8	Fine sand, I_D = 0.56	33.5	5	72
9	Fine sand, I_D = 0.78	36	5	81
10	Stone columns	35	10	100

7. CONCLUSIONS

Based on analyses performed in the article it is possible to state with an absolute certainty that the proposed estimation method for bearing capacity of the subsoil may be very effective in many problems of soil engineering. This method is a generalisation of numerical procedure of stability analysis. Also in this case a failure mechanism is sought after (e.g. soil displacement from under the foundation) and a corresponding stability coefficient. In the analysis method described in the paper, knowing this coefficient value it is possible to estimate the subsoil bearing capacity at any arrangement of soil layers in the subsoil and at any way of loading. Moreover, the method is very simple and available in numerous packages of FEM systems used at designing.

Performed numerical tests have proved that the presented method gives results convergent with those obtained based on analytical formulae used so far. So it is possible to assess that it provides reliable results in a wide range of applications. Moreover, it is equally important that the obtained bearing capacity of the subsoil results for consideration of the failure mechanism resulting from numerical calculations (here, with the use of FEM) and not based on making some assumptions. Therefore the obtained results may be considered reliable and the method is recommendable for use in practical applications.

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