# SOLAR & WIND HYBRID POWER SOURCE FOR RESIDENTIAL BUILDING MATHEMATICAL MODEL APPROACH

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Received: 11.06.2015; Revised: 20.06.2015; Accepted: 10.11.2015

Abstract

**Solar and wind are clean energy sources with capability of decreasing grid dependence. The aim of this paper was to present them as possible sources of an electrical energy in a residential building. To roll out principles and realize given objective functions of a hybrid power system a mathematical model has been developed. Conducted calculations have shown that the solar energy is more stable and predictable energy source than wind, however the combination of both allows reduction in capacity of a battery bank.**

Streszczenie

**Słońce i wiatr to odnawialne źródła czystej energii, które pozwalają na zmniejszenie uzależnienia od sieci elektroenergetycznej lub uzyskanie całkowitej samowystarczalności energetycznej. Celem poniższego artykułu było przedstawienie ich, jako możliwych źródeł energii elektrycznej w budynku mieszkalnym. W celu zaprezentowania zasad funkcjonowania systemu hybrydowego zbudowany został model matematyczny, który jako odwzorowanie rzeczywistości pozwala na przeprowadzenie szeregu eksperymentów i symulacji na podstawie wcześniej przyjętych ograniczeń i funkcji celu. Wykonane na jego podstawie obliczenia wykazały, iż źródło energii wykorzystujące promieniowanie słoneczne jest stabilniejsze niż elektrownie wiatrowe. Jednakże połączenie obu tych źródeł energii elektrycznej ze względu na ich czasową komplementarność pozwala na ograniczenie niezbędnej pojemności akumulatorów.**

Keywords: **Hybrid power system; Renewable energy; Energy storage; Modelling; Optimization.**

## 1. INTRODUCTION

The world total energy consumption over years 1990 – 2007 has increased by nearly 40% and it is highly probable that this growth will be continued till the year 2035 whilst experiencing an average increase between 8 to 10% every 5 years. This will be partially a result of an increase in population, rapid urbanization and development of industry and societies of other countries of the so called third world [1]. According to the Building Energy Data Book [2] over 40% of energy consumption in the United States is attributable to the construction sector, thereby exceeding demand created separately by industry and transport sectors. From a global point of view buildings contribute (similarly as in USA) to 40% consumption in terms of a primary energy, but taking into the consideration energy used in manufacturing processes of steel, cement, glass or aluminum this share would increase to 50% [3]. Throughout the year 2012 renewable energy sources have covered 19% of the world demand for a final energy [4] this quota will remain at a stable level despite significant amount of new investments in RES. According to the *Polish Information Agency and Foreign Investments* [5] demand for the final energy in individual sectors of the

Polish economy until the year 2020 will be as follows [Mtoe]: industry – 20.9, transport – 18.7, agriculture – 5, services – 8.8, households – 19.4. It is estimated by the Polish Government that over years 2010 – 2020 an average consumption of a primary energy will increase by 1.5% per year, however in the same time the share of an energy from renewable energy sources should achieve the level of 12% [6].

## 2. SOLAR AND WIND ENERGY

Renewable energy sources compared to conventional ones are characterized by relatively low concentration of electrical or heat power possible to be generated from unit of energy source volume or surface. Therefore it means that acquisition of similar power output to conventional energy sources may require devices and installations of much greater dimensions. This in case of solar energy means the surface designated for photovoltaic modules or heliostats – which concentrate direct solar radiation on given point. In Poland with the current efficiency of photovoltaic modules and the whole system of energy conversion, from 6 $m^2$ of monocrystalline modules one may generate approximately 1 MWh of electrical energy per year [7]. By contrast in wind energy this forces developers to locate wind turbines on considerable height which allows to obtain greater stability in terms of generated electrical energy and to increase the possible power output of single turbines. The wind map for Poland clearly shows that distribution of yearly mean wind speed in uneven. In effect it is possible to distinguish five zones on the ground of their fitness for the purpose of the wind energy. It is of importance to note that this map is only the first approximation and a selection of site (especially for a large wind park) should be preceded by at least yearly measurements of wind speed.

The aim of this paper was to analyze the possibility of harnessing wind and solar radiation energy in order to supply household with electrical energy. It is worth mentioning that the interest in hybrid power generators results mainly from partial temporal complementarity of solar and wind resources, which enables the creation of energy source which operates stable in real time. The analysis of that question belonged to the area of research of inter alia: *Monforti F. et. al*. [8], *Santos-Alamillos F.J.et. al.* [9] oraz *Jerez S. et. al.* [10]. Results of studies conduced in the mentioned papers find applications as a part of public utility power stations where rated capacity of renewable energy sources in counted in tens or hundreds of megawatts. From the point of view of small hybrid power source, which will be utilized for the need of detached house, not only temporal complementarity of wind and solar resources is of importance but in particular their availability on limited space. This stakes out mainly to wind energy, on the grounds of minimal wind speed at which the wind turbine will start generating power, the so called cut-in speed. What is more each wind turbine has its own power generation curve which is strongly dependent on wind speed. In this work an hourly measurements of wind speed at 10 meters for year 2004 from meteorological station in Toruń-Wrzosy were used. On the other hand the amount of solar radiation energy can be exactly calculated based on clear sky model [11]. But all deviations from calculated values should be accounted to changes of the following parameters: local weather conditions and contents of compounds and substances which disperse solar radiation. In this study data concerning solar radiation falling on 30° inclined surface were obtained from www.soda-is.com [12].

## 3. MATHEMATICAL MODEL

A mathematical model of a hybrid power source equipped with energy storage (battery bank) allows sizing individual parts of the system in such a way that the assumed objective function will be realized and constraints fulfilled. It is important to note that model is a simplification of a reality. It enables understanding of dependencies which exist in real system and allows conducting experiments which with certain degree of accuracy will reproduce the behavior of real one.

### 3.1. Model variables and parameters

In the mathematical model following variables and parameters describing behavior of its individual elements have been used. At the modeling stage values of mentioned parameters were obtained from supplier's websites [14, 15, 16, 17].

$I = (1, \ldots, I) - set\ of\ days,$

$J = (1, \ldots, J) - set\ of\ hours,$

$K = (1, \ldots, K) - set\ of\ wind\ turbines\ output\ power,$

$L = (1, \ldots, L) - set\ of\ wind\ turbine\ hub\ height,$

$P - nominal\ power\ o\ PV\ system,$

$WW - coefficient\ of\ PV\ system\ efficiency,$

$x_{i,j} - irrdiation\ during\ day\ i\ and\ hour\ j,$

$Z_{i,j} - energy\ consumption\ during\ day\ i\ and\ hour\ j,$

$S_{i,j} - energy\ yield\ from\ Sun\ during\ day\ i\ and\ hour\ j,$

$W_{i,j,k,l} - energy\ yield\ from\ wind\ during\ day\ i\ and\ hour\ j\ from\ wind\ turbine\ of\ power\ k\ at\ height\ l,$

$SW_{i,j} - total\ energy\ yield\ during\ day\ i\ and\ hour\ j,$

$KW_{k,l} - cost\ of\ wind\ turbine\ of\ power\ k\ at\ height\ l,$

$KEW - total\ cost\ of\ wind\ turbines,$

$KPV - cost\ of\ photovoltaic\ system,$

$c_1 - pruchase\ price\ of\ 1\ kWh\ from\ the\ grid,$

$c_2 - resale\ price\ of\ 1\ kWh\ to\ the\ grid,$

$c_2 - cost\ of\ 1\ kWh\ electrical\ energy\ storage\ in\ battery\ bank$

$a_{k,l} - decision\ variable, wind\ turbines,$

$TW - number\ of\ wind\ turbines,$

$DE_{i,j} - energy\ deficite\ during\ day\ i\ and\ hour\ j,$

$Def - permissible\ energy\ deficite\ ,$

$SA_{i,j} - battery\ bank\ state\ of\ charge\ during\ day\ i\ and\ hour\ j,$

$SPA - intitial\ battery\ bank\ state\ of\ charege\ ,$

$PA - nameplate\ battery\ bank\ capacity,$

$KA - battery\ bank\ cost,$

$KCS - total\ cost\ of\ electrical\ energy\ supply\ system,$

$NF - maximal\ expenditure\ on\ electrical\ energy\ supply\ system,$

$AF_{i,j} - income\ from\ savings\ or\ energy\ resale\ during\ day\ i\ and\ hour\ j,$

**3.2. Equations describing power supply, generation and energy cost variability**

By means of equations (2a-g) a principle has been described according to which: an energy deficiency and its yield for individual energy sources as well as corresponding costs of their implementation to the system have been calculated.

$$DE_{i,j} = Z_{i,j} - SW_{i,j} \quad -energy\ deficiency\ during\ day\ i\ at\ hour\ j \tag{1a}$$

$$a_{k,l} = \begin{cases} 1\ if\ wind\ turbine\ of\ nominal\ power\ k\ at\ hight\ l\ has\ been\ selected \\ 0, otherwise \end{cases} \tag{1b}$$

$$KEW = \sum_{k\in K}\sum_{l\in L} a_{k,l} * KW_{k,l} \quad -total\ cost\ of\ wind\ turbines \tag{1c}$$

$$KPV = f(P) = -0.23P^3 + 16.6P^2 - 430.3P + 6473.6 \quad for\ P \in \langle 1; 40\rangle \tag{1d}$$

$$KA = c_3 * PA \tag{1e}$$

$$S_{i,j} = P * WW * x_{i,j} \quad -energy\ yield\ from\ PV\ during\ day\ i\ at\ hour\ j \tag{1f}$$

$$SW_{i,j} = S_{i,j} + a_{k,l} * W_{i,j,k,l} \tag{1g}$$

**3.3. Energy storage – battery charge**

In the presented model an assumption has been made that each emerging surplus in terms of electrical energy which has not been used to cover present energy demand will be stored in battery bank of given nominal capacity. If over individual hour more energy is generated than it will be possible to use or store it rather than sell this excessive energy to the grid. This rule has been described by equation (2):

$$SA_{i,j} = \begin{cases} SA_{i,j-1} + DE_{i,j} > PA\ to\ PA \\ if\ SA_{i,j-1} + DE_{i,j} < PA\ i > \ 0\ to\ SA_{i,j-1} + DE_{i,j} \\ SA_{i,j-1} + DE_{i,j} < 0\ to\ 0 \end{cases} \tag{2}$$

**3.4. Savings and resale revenue**

The unused and not stored in battery bank electrical energy may be resold to the mains for at earlier fixed price. The electrical energy which was not sold and was used to fulfill owner needs is perceived as a source of savings resulting from no need to buy it from the electrical grid. The formula for calculating the savings and resale revenue is given by means of equations (3a-b).

$$\sum_{i\in I}\sum_{j\in J} AF_{i,j} = \sum_{i\in I}\sum_{j\in J} c_1 * SW_{i,j} + \sum_{i\in I}\sum_{j\in J} c_1 * Z_{i,j} + \sum_{i\in I}\sum_{j\in J} (c_1 * Z_{i,j} + c_2 * \left|Z_{i,j} - SW_{i,j}\right|)\ (3a) \qquad (3a)$$

Subject to:

$$if\ DE_{i,j} \begin{cases} > 0, to\ c_1 * SW_{i,j} \\ = 0, to\ c_1 * Z_{i,j} \\ < 0, to\ c_1 * Z_{i,j} + c_2 * \left|Z_{i,j} - SW_{i,j}\right| \end{cases} \qquad (3b)$$

### 3.5. Savings and resale revenue

The total cost of a hybrid power source is a sum of its individual parts, which can be expressed by formula (4), whereas its components are calculated based on (1c-e).

$$KCS = KPV + KEW + KA \qquad (4)$$

### 3.6. Objective function

Created mathematical model enables implementation of different objective functions. As a part of this paper it has been assumed that the following will be realized: minimization of total cost of the system (5a), minimization of cost of unit of electrical energy based on 10 years operation period of installation (5b) and minimization of electrical energy deficiency (5c).

$$min\ KCS = KPV + KEW + KA \qquad (5a)$$

$$min \frac{\frac{1}{3}KPV + \frac{1}{2}KEW + KA}{10\sum_{i\in I}\sum_{j\in J} AF_{i,j}} \qquad (5b)$$

$$\min \sum_{i\in I}\sum_{j\in J} DE_{i,j} \qquad (5c)$$

### 3.7. Constraints

Each objective function (5a-c) will be realized under the provision of certain initial model parameters which relate to inter alia purchase cost of electricity, storage cost and permissible investment outlay. In each variant of the presented model a constraint has been made concerning the maximal number of installed wind turbines (6a). The next constrain is the maximal deficiency of electrical energy which may occur and has to be covered by the energy from the mains (6b). It has been acknowledged that there is a possibility to limit the total investment outlay (6c) and the permissible capacity of battery bank (6d).

$$\sum_{k\in K}\sum_{l\in L} a_{k,l} \leq TW \qquad (6a)$$

$$\sum_{i\in I}\sum_{j\in J} DE_{i,j} \leq Def \qquad (6b)$$

$$KCS \leq NF \qquad (6c)$$

$$PA \leq 20 \qquad (6d)$$

### 3.8. Model variants

The optimization oriented on the finding the most beneficial value of objective function have been conducted for three variants of the presented model. Their specification has been presented in Table 1. In all cases: $c_1 = 0.56$ $c_2 = 0.15$ $c_3 = 500$ TW = 2.

a) minimization of total cost (investment outlay), with constraint to the total amount of energy acquired from the mains;
b) maximization of the revenue from the installation;
c) coverage of total electrical energy demand, whereas the power system total cost is limited.

**Table 1.**
**Individual variants**

| Variant | a | b | c |
|---|---|---|---|
| Obj. Fun. | 5a | 5b | 5c |
| Constraints | 6a,6b, 6d | 6a,6b, 6d | 6a, 6c |
| Parameters | Def=10 | X | NF=56000 |

## 4. CASE STUDY

The created mathematical model has been used to simulate the behavior of hybrid power source supplying household which consumes on a yearly basis 4500 kWh of electrical energy. In simulation a simplification has been made such that every day is described by the same energy demand curve, presented in Figure 1.

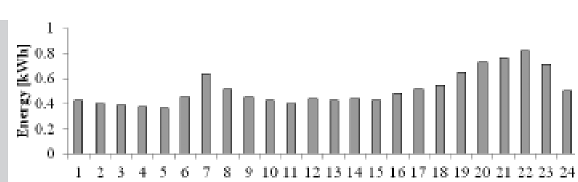


**Figure 1.**
**Electrical energy consumption profile**

Whereas Figure 2 presents a distribution of an hourly mean wind speed at 10 meters above ground. Wind speed not only fluctuates from hour to hour but also on longer time scale, what has been depicted in Figure 3. Juxtaposing these values with those presented in Figure 4 one may come to a conclusion that to a certain degree wind energy and sun energy complement each other.

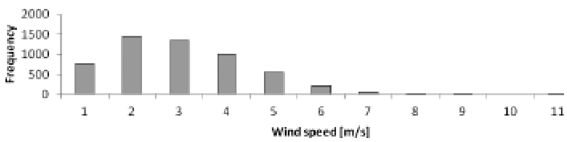


**Figure 2.**
**Histogram for yearly mean wind speed based on measurements from meteorological station Toruń-Wrzosy (2004)**

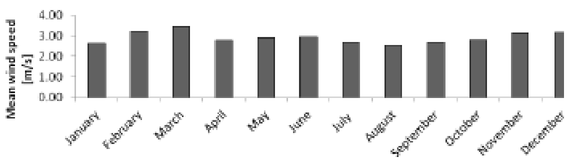


**Figure 3.**
**Monthly mean wind speed in year 2004**

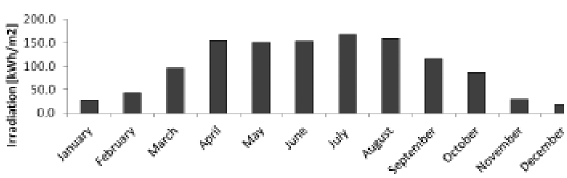


**Figure 4.**
**Monthly sum of irradiation in year 2004 based on [12]**

## 5. RESULTS

Simulation of hybrid power source have been done in MS Excel 2013 using data analysis tool named Solver. Obtained results are presented in Table 2. Besides presenting only objective function values for each variant an installed power in each source, battery bank capacity, investment outlay for each component and the energy balance have been given.

The variability of solar radiation and wind speed is a determinant factor influencing the energy balance. In Figure 5 and 6 a hybrid power system energy production, storage and consumption from scenario "c" for two succeeding days have been presented. Those days refer to January and late October.

**Table 2.**
**Parameters values after optimization procedure**

| Scenario | a | b | c |
|---|---|---|---|
| **Objective Function** | 177843 | 0.86 | 387.8 |
| **KCS [PLN]** | = *Objective function* | 192720 | 56000 |
| **KPV [PLN]** | 138530 | 187776 | 21905 |
| **KEW [PLN]** | 32900 | 0 | 25800 |
| **KA [PLN]** | 6412 | 4943 | 8294 |
| **PV [kW]** | 24.1 | 40 | 4 |
| **PA [kWh]** | 12.8 | 9.9 | 16.6 |
| **TW [kW] \| [m]** | 0.3\|15 3\|20 | 0 | 5\|10 |
| **Demand covered [kWh]** | 4491 | 4490 | 4112 |
| **Energy resold [kWh]** | 25250 | 35460 | 2218 |
| **Energy bought from the grid [kWh]** | 8.6 | 10 | =*Objective function* |

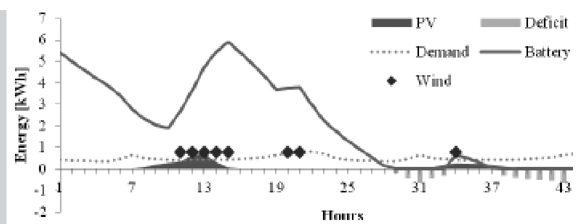


**Figure 5.**
**Energy balance for two days in January**

As it can be seen popping up intermittent outburst of wind allowed covering to some extend energy demand. However, all four days were rather cloudy therefore the total energy yield from photovoltaic installation was not enough in comparison with the energy demand. Only during the first day in Fig. 4 the energy generation from PV was greater than energy demand, as a result an increase in the amount of energy stored in battery bank can be observed.

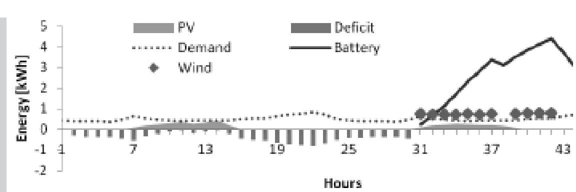


**Figure 6.**
**Energy balance for two days in October**

It is of importance to note that in presented models there was not limit with regard to the discharge state of the battery. Therefore in order to protect it from too deep discharge it would be advisable to increase its capacity or install an adequate sensor.

## 5. CONCLUSIONS

Presented in this paper mathematical model allows decision makers to realize chosen objective functions which should be realized while complying with constraints. This model is fully functional in MS Excel software what should be perceived as an asset because of its ease to be modified and implemented by almost everyone. The main advantage of proposed approach is the possibility to look at the changes in electrical energy balance over selected periods. Application as input data hourly values of energy generation and demand makes this model very close to the reality. Future study should focus on using real life hourly energy demand data and on expanding issues relating to the operation of the battery bank.

## ACKNOWLEDGEMENT

This paper in an extended form has been presented during BIWA Conference held in Gliwice (23-24.05.2015 – Poland) and will be published as a part of a post conference monograph in Polish.

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